

1 Linear Algebra

$$\begin{aligned}\vec{v} + \vec{w} &= (v_1 + w_1)\hat{i} + (v_2 + w_2)\hat{j} + (v_3 + w_3)\hat{k} \\ \vec{v} \cdot \vec{w} &= v_1w_1 + v_2w_2 + v_3w_3 = |\vec{v}||\vec{w}|\cos\theta \\ \vec{v} \cdot \vec{w}_{\perp\vec{v}} &= 0 \\ \vec{v} \times \vec{w} &= (v_2w_3 - v_3w_2)\hat{i} - (v_1w_3 - v_3w_1)\hat{j} + (v_1w_2 - v_2w_1)\hat{k} \\ \vec{v} \times \vec{w} &= \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix} = -(\vec{w} \times \vec{v}) \\ \vec{v} \times (\vec{w} \parallel \vec{v}) &= 0 \quad |\vec{v} \times \vec{w}| = |\vec{v}||\vec{w}|\sin\theta \\ \vec{u} = \frac{\vec{v}}{|\vec{v}|} \quad \hat{i} \times \hat{j} &= \hat{k} \quad \hat{j} \times \hat{k} = \hat{i} \quad \hat{k} \times \hat{i} = \hat{j} \\ |\vec{v}| &= \sqrt{v_1^2 + v_2^2 + v_3^2} \\ \vec{v} \parallel \vec{w} \Rightarrow \vec{w} &= \lambda \vec{v} \\ \mathbf{M} &= \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad \mathbf{M}^{-1} = \frac{1}{\det \mathbf{M}} \begin{bmatrix} m_{22} & -m_{12} \\ -m_{21} & m_{11} \end{bmatrix} \\ \det \mathbf{M} &= m_{11}m_{22} - m_{12}m_{21}\end{aligned}$$

1.1 Projections ($\vec{v}$ onto $\vec{w}$)

$$\begin{aligned}(\vec{v} \parallel \vec{w}): \text{Scalar: } v' &= |\vec{v}|\cos\theta \quad \text{Vector: } \vec{v}' = v'\hat{w} = \vec{v} \cdot \frac{\vec{w}}{|\vec{w}|} \\ (\vec{v} \perp \vec{w}): \text{Rejection: } \vec{v}'' &= \vec{v} - \vec{v}'\end{aligned}$$

1.2 Plane with Normal $\vec{n}$ and Point P_0

$$\begin{aligned}\vec{n} &= a\hat{i} + b\hat{j} + c\hat{k} \quad P_0 = (x_0, y_0, z_0) \\ a(x - x_0) + b(y - y_0) + c(z - z_0) &= 0 \Rightarrow ax_0 + by_0 + cz_0 = d \\ ax + by + cz &= d\end{aligned}$$

1.3 Plane Passing Through 3 Points, $\angle RPQ$

$$\begin{aligned}P &= (p_1, p_2, p_3) \quad Q = (q_1, q_2, q_3) \quad R = (r_1, r_2, r_3) \\ \overrightarrow{PQ} &= \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle \quad \overrightarrow{PR} = \langle r_1 - p_1, r_2 - p_2, r_3 - p_3 \rangle \\ \overrightarrow{PQ} \times \overrightarrow{PR} &= a\hat{i} + b\hat{j} + c\hat{k} \Rightarrow a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \\ &\text{with } x_0, y_0, z_0 \text{ from } P, Q, \text{ or } R\end{aligned}$$

2 Differential Equations

$$\begin{aligned}\frac{dy}{dx} &= x \rightarrow y = \frac{1}{2}x^2 + C \\ \frac{dP}{dt} &= kP \rightarrow P = P_0 e^{kt} \quad k \in \mathbb{R} \\ \frac{dT}{dt} &= k(T_0 - T_E) \rightarrow T = (T_0 - T_E)e^{kt} + T_E \\ \text{Product rule: } h(x) &= f(x)g(x) \rightarrow h'(x) = f'(x)g(x) + f(x)g'(x) \\ \text{Quotient rule: } h(x) &= \frac{f(x)}{g(x)} \rightarrow h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \\ \text{Chain rule: } h(x) &= f(g(x)) \rightarrow h'(x) = f'(g(x)) \cdot g'(x) \\ \text{Equivalently: } \frac{dz}{dx} &= \frac{dz}{dy} \cdot \frac{dy}{dx} \\ \text{Separation of variables: } \frac{dy}{dx} &= f(x) \cdot g(y) \rightarrow \frac{1}{g(y)} dy = f(x)dx\end{aligned}$$

2.1 Identities

$$\begin{aligned}\frac{d}{dx} e^{f(x)} &= f'(x)e^{f(x)} \quad \frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)} \\ \frac{d}{dx} a^x &= a^x \ln a \quad \frac{d}{dx} \log_a x = \frac{1}{x \ln a} \quad x > 0\end{aligned}$$

2.2 Partial Differential Equations: $f(x, y, z)$

$$\begin{aligned}\text{Grad: } \nabla f &= \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k} \quad \nabla_{\vec{v}} f = \vec{v} \cdot \nabla f \\ \text{Div: } \nabla \cdot f &= \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} \\ \text{Curl: } \nabla \times f &= \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{bmatrix} \\ \text{Chain rule: } z &\equiv f(x(t), y(t)) : \frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} \\ z &\equiv f(x_1(t_1, \dots, t_m), \dots, x_n(t_1, \dots, t_m)) : \\ \frac{\partial z}{\partial t_i} &= \frac{\partial z}{\partial x_1} \frac{\partial x_1}{\partial t_i} + \dots + \frac{\partial z}{\partial x_n} \frac{\partial x_n}{\partial t_i}\end{aligned}$$

3 Integration

$$\begin{aligned}\text{By parts: } \int u dv &= uv - \int v du \\ \text{or: } \int u \left(\frac{dv}{dx}\right) dx &= uv - \int v \left(\frac{du}{dx}\right) dx \\ \text{Substitution rule: } \int f(u)u' dx &= \int f(u) du\end{aligned}$$

$$\begin{aligned}\text{Integrating factor given linear D. E., } \frac{dy}{dt} + P(t)y &= Q(t): \\ \mu(t) &= e^{\int P(t) dt} \quad \int \frac{d}{dt}(\mu(t) \cdot y) dt = \int \mu(t)Q(t) dt\end{aligned}$$

3.1 Identities

$$\int e^x dx = e^x \quad \int e^{-x} dx = -e^{-x} \quad \int e^{ax} dx = \frac{1}{a}e^{ax} \quad \int a^x dx = \frac{a^x}{\ln a}$$

3.2 Applications

$$\begin{aligned}\text{Disk, } \odot x: V &= \pi \int_a^b [f(x)]^2 dx \\ A_S &= 2\pi \int_a^b f(x) \sqrt{1 + [f'(x)]^2} dx \\ \text{Washer, } \odot x: V &= \pi \int_a^b [f(x)]^2 - [g(x)]^2 dx \\ \text{Shell, } \odot x: V &= 2\pi \int_c^d yg(y) dy \\ \text{Shell, } \odot y: V &= 2\pi \int_a^b xf(x) dx \\ \ell_a &= \int_{P_1}^{P_2} \sqrt{1 + [f'(x)]^2} dx \\ x = f(t), y = g(t) \rightarrow \ell_a &= \int_{P_1}^{P_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt\end{aligned}$$

$$A_S = \iint_S \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + 1} dxdy$$

3.3 Partial Fractions: $f(x) = P(x)/Q(x)$

$$Q(x) : (ax+b)^k \rightarrow \sum_{i=1}^k \frac{A_i}{(ax+b)^i} \quad (ax^2+bx+c)^k \rightarrow \sum_{i=1}^k \frac{A_i x + B_i}{(ax^2+bx+c)^i}$$

3.4 Line Integral

$$\begin{aligned}\text{Line integral:} \\ \vec{F} &= \langle F_1\hat{i}, F_2\hat{j}, F_3\hat{k} \rangle \quad \vec{r} = \langle x\hat{i}, y\hat{j}, z\hat{k} \rangle \\ \int_C \vec{F} \cdot d\vec{r} &= \int_{P_1}^{P_2} \vec{F}(\vec{r}(t)) \cdot \vec{r}' dt = \int_C (F_1 dx + F_2 dy + F_3 dz) \\ \text{Conservative field, endpoints } P_1, P_2: \\ \vec{F} = \nabla f \quad \int_C \vec{F} \cdot d\vec{r} &= \int_{P_1}^{P_2} \nabla f \cdot d\vec{r} = f(P_2) - f(P_1) \\ \therefore \int_{C_n} \vec{F} \cdot d\vec{r} &= \int_{C_{m \neq n}} \vec{F} \cdot d\vec{r}\end{aligned}$$

$$\text{Flux integral: } \Phi = \iint_S \vec{F} \cdot \hat{n} dS$$

$$\text{Divergence theorem: } \oint_S \vec{F} \cdot \hat{n} dS = \iiint_V (\nabla \cdot \vec{F}) dV$$

Green's theorem:

$$\begin{aligned}\vec{F}(x, y) &= P(x, y)\hat{i} + Q(x, y)\hat{j} \\ \oint_C \vec{F} \cdot d\vec{r} &= \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA\end{aligned}$$

$$\text{Stokes' theorem: } \oint_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

4 Series

$$\begin{aligned}\text{Arithmetic: } \sum_{k=0}^{n-1} a + kd &= \frac{n}{2}(2a + (n-1)d) \\ \text{Geometric: } \sum_{k=0}^{n-1} ar^k &= a \left(\frac{r^n - 1}{r - 1}\right)\end{aligned}$$

5 Statistics

$$\begin{aligned}\bar{x} &= \frac{\sum x_i}{n} \quad \mu = \frac{\sum x_i}{N} \quad s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \quad \sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}} \\ r_{xy} &= \frac{1}{n-1} \sum \frac{x_i - \bar{x}}{s_x} \frac{y_i - \bar{y}}{s_y} \\ \text{Sample geometric mean: } \sqrt[n]{\prod x_i} \quad \text{Sample RMS: } \sqrt{\frac{\sum x_i^2}{n}}\end{aligned}$$

5.1 Binomials

$$\begin{aligned}(a+b)^n &= \sum_{k=0}^n \binom{n}{k} a^k b^{n-k} \quad \binom{n}{k} = \frac{n!}{k!(n-k)!} \\ n \in \mathbb{N} \quad k \in \mathbb{N}^* \quad k &\leq n\end{aligned}$$

5.2 Probability and Combinatorics

$$\begin{aligned} \mathbb{P}(A \vee B) &= \mathbb{P}(A) + \mathbb{P}(B) & \mathbb{P}(A \cup B) &= \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) \\ \mathbb{P}(A \wedge B) &= \mathbb{P}(A)\mathbb{P}(B) & \mathbb{P}(A \cap B) &= \mathbb{P}(A|B)\mathbb{P}(B) = \mathbb{P}(B|A)\mathbb{P}(A) \\ \mathbb{P}(A|B) &= \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B|A)\mathbb{P}(A) + \mathbb{P}(B|\neg A)\mathbb{P}(\neg A)} \end{aligned}$$

Binomial distribution: k successes, n trials, success probability $\mathbb{P}$:

$$\begin{aligned} B(k; n, \mathbb{P}) &= \binom{n}{k} \mathbb{P}^k (1 - \mathbb{P})^{n-k} \\ {}^n P_k &= n^k \text{ (repetition)} & {}^n P_k &= \frac{n!}{(n-k)!} \text{ (no repetition)} \\ {}^n C_k &= \frac{(n+k-1)!}{k!(n-1)!} \text{ (repetition)} & {}^n C_k &= \frac{n!}{k!(n-k)!} \text{ (no repetition)} \end{aligned}$$

$$\text{Multisets: } {}^n P_r(n; n_1, n_2, \dots, n_k) = \frac{n!}{n_1! n_2! \dots n_k!}$$

$$\text{Derangement: } !n = n! \sum_{i=0}^n \frac{(-1)^i}{i!} \quad n \geq 0$$

5.3 t-Statistic

$$\text{One sample: } t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Paired samples:

$$t = \frac{\sum (x_1 - x'_1)}{n} \left[\frac{\sum (x_1 - x'_1)^2 - \frac{[\sum (x_1 - x'_1)]^2}{n}}{n(n-1)} \right]^{-1/2}$$

Independent samples:

$$t = (\mu_A - \mu_B) \left[\left(\frac{\sum (A^2) - \frac{(\sum A)^2}{n_A} + \sum (B^2) - \frac{(\sum B)^2}{n_B}}{n_A + n_B - 2} \right) \left(\frac{1}{n_A} + \frac{1}{n_B} \right) \right]^{-1/2}$$

5.4 Statistical Tests for Comparing Means

$$\frac{s_1^2}{s_2^2} > F. \text{ INV. RT } (\alpha, \nu_1, \nu_2) \Rightarrow \text{significant}$$

$$s_m \sim s_n : t = \frac{|\bar{x}_1 - \bar{x}_2|}{s_{\text{pooled}}} \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \quad s_{\text{pooled}} = \sqrt{\frac{s_1^2 \nu_1 + s_2^2 \nu_2}{\nu_1 + \nu_2}}$$

$$s_m \approx s_n : t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$$

$$\nu' = \frac{[s_1^2/n_1 + s_2^2/n_2]^2}{(s_1^2/n_1)^2/\nu_1 + (s_2^2/n_2)^2/\nu_2} \quad t > T. \text{ INV. } 2T(\alpha, \nu') \Rightarrow \text{significant}$$

6 Geometry

$$\begin{aligned} A_{\text{parallelogram}} &= bh & A_{\text{triangle}} &= 0.5bh \\ A_{\text{trapezoid}} &= 0.5(a+b)h & A_{\text{circle}} &= \pi r^2 \\ V_{\text{cube}} &= s^3 & A_{s, \text{cube}} &= 6s^2 \\ V_{\text{sphere}} &= \frac{4}{3}\pi r^3 & A_{s, \text{sphere}} &= 4\pi r^2 \\ V_{\text{rec. prism}} &= \ell_1 \ell_2 h & A_{s, \text{rec. prism}} &= 2(\ell_1 \ell_2 + \ell_1 h + \ell_2 h) \\ V_{\text{cylinder}} &= \pi r^2 h & A_{s, \text{cylinder}} &= 2\pi r h + 2\pi r^2 \\ V_{\text{rt. circ. cone}} &= \frac{\pi r^2 h}{3} & A_{s, \text{rt. circ. cone}} &= \pi r \sqrt{r^2 + h^2} \\ V_{\text{pyramid}} &= \frac{\ell_1 \ell_2 h}{3} \\ A_{s, \text{pyramid}} &= \ell_1 \ell_2 + \ell_1 \sqrt{\left(\frac{\ell_2}{2}\right)^2 + h^2} + \ell_2 \sqrt{\left(\frac{\ell_1}{2}\right)^2 + h^2} \end{aligned}$$

7 Trigonometric Identities

$$\sin \theta = \csc^{-1} \theta \quad \cos \theta = \sec^{-1} \theta \quad \tan \theta = \cot^{-1} \theta$$

$$\text{co-function}(\frac{\pi}{2} - \theta) = \text{function}(\theta)$$

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \quad \cos^2 \theta = \frac{1 + \cos 2\theta}{2} \quad \tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$\sin u + \sin v = 2 \sin \frac{u+v}{2} \cos \frac{u-v}{2}$$

$$\sin u - \sin v = 2 \cos \frac{u+v}{2} \sin \frac{u-v}{2}$$

$$\cos u + \cos v = 2 \cos \frac{u+v}{2} \cos \frac{u-v}{2}$$

$$\cos u - \cos v = -2 \sin \frac{u+v}{2} \sin \frac{u-v}{2}$$

$$\sin u \sin v = \frac{\cos(u-v) - \cos(u+v)}{2}$$

$$\cos u \cos v = \frac{\cos(u-v) + \cos(u+v)}{2}$$

$$\sin u \cos v = \frac{\sin(u+v) + \sin(u-v)}{2}$$

$$\cos u \sin v = \frac{\sin(u+v) - \sin(u-v)}{2}$$

$$\cos^2 \theta + \sin^2 \theta = 1 \quad e^{i\pi} + 1 = 0 \quad e^{ix} = \cos x + i \sin x$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \sin x = \frac{e^{ix} - e^{-ix}}{2}$$

8 Fourier and Laplace Transforms

$$\mathcal{F}g(t) = \hat{g}(f) = \int_{-\infty}^{\infty} g(t) e^{-2\pi i f t} dt$$

$$\mathcal{F}^{-1}\hat{g}(f) = g(t) = \int_{-\infty}^{\infty} \hat{g}(f) e^{2\pi i f t} df$$

$$f(t) \sim A_0 + \sum_{n=1}^{\infty} [A_n \cos(n \frac{2\pi t}{T}) + B_n \sin(n \frac{2\pi t}{T})]$$

$$A_0 = \frac{1}{T} \int_{-T/2}^{T/2} f(t) dt$$

$$A_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(n \frac{2\pi t}{T}) dt \quad B_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin(n \frac{2\pi t}{T}) dt$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{-n2\pi i t}{T}} \quad c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{\frac{-n2\pi i t}{T}} dt \quad n \in \mathbb{Z}$$

$$\mathcal{L}g(t) = G(s) = \int_0^{\infty} g(t) e^{-st} dt \quad s = \sigma + i\omega \quad 0 < t < \infty$$

$$\mathcal{L}^{-1}G(s) = g(t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} G(s) e^{st} ds$$