1 Linear Algebra

$\vec{v}+\vec{w}=(v_1+w_1)\hat{\imath}+(v_2+w_2)\hat{\jmath}+(v_3+w_3)\hat{k}$
$\vec{v}\cdot\vec{w}=v_1w_1+v_2w_2+v_3w_3=|\vec{v}||\vec{w}|\cos\theta$
$\vec{v}\cdot\vec{w}_{\perp\vec{v}}=0$
$\vec{v}\times\vec{w}=(v_2w_3-v_3w_2)\hat{\imath}-(v_1w_3-v_3w_1)\hat{\jmath}+(v_1w_2-v_2w_1)\hat{k}$

$$\vec{v}\times\vec{w}=\det\begin{bmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\ v_1&v_2&v_3\\ w_1&w_2&w_3\end{bmatrix}=-(\vec{w}\times\vec{v})$$

$\vec{v}\times(\vec{w}\parallel\vec{v})=0 \qquad |\vec{v}\times\vec{w}|=|\vec{v}||\vec{w}|\sin\theta$
$\vec{u}=\frac{\vec{v}}{|\vec{v}|} \qquad \hat{\imath}\times\hat{\jmath}=\hat{k} \qquad \hat{\jmath}\times\hat{k}=\hat{\imath} \qquad \hat{k}\times\hat{\imath}=\hat{\jmath}$
$|\vec{v}|=\sqrt{v_1^2+v_2^2+v_3^2}$
$\vec{v}\parallel\vec{w}\Rightarrow\vec{w}=\lambda\vec{v}$

$$\boldsymbol{M}=\begin{bmatrix}m_{11}&m_{12}\\ m_{21}&m_{22}\end{bmatrix} \qquad \boldsymbol{M}^{-1}=\frac{1}{\det\boldsymbol{M}}\begin{bmatrix}m_{22}&-m_{12}\\ -m_{21}&m_{11}\end{bmatrix}$$

$\det\boldsymbol{M}=m_{11}m_{22}-m_{12}m_{21}$

1.1 Projections ($\vec{v}$ onto $\vec{w}$)

$(\vec{v}\parallel\hat{w})$: Scalar: $v'=|\vec{v}|\cos\theta$ Vector: $\vec{v}'=v'\hat{w}=\vec{v}\cdot\frac{\vec{w}}{|\vec{w}|}$
$(\vec{v}\perp\hat{w})$: Rejection: $\vec{v}''=\vec{v}-\vec{v}'$

1.2 Plane with Normal $\vec{n}$ and Point P_0

$\vec{n}=a\hat{\imath}+b\hat{\jmath}+c\hat{k} \qquad P_0=(x_0,y_0,z_0)$
$a(x-x_0)+b(y-y_0)+c(z-z_0)=0\Rightarrow ax_0+by_0+cz_0=d$
$ax+by+cz=d$

1.3 Plane Passing Through 3 Points, $\angle RPQ$

$P=(p_1,p_2,p_3) \quad Q=(q_1,q_2,q_3) \quad R=(r_1,r_2,r_3)$
$\overrightarrow{PQ}=\langle q_1-p_1,q_2-p_2,q_3-p_3\rangle \qquad \overrightarrow{PR}=\langle r_1-p_1,r_2-p_2,r_3-p_3\rangle$
$\overrightarrow{PQ}\times\overrightarrow{PR}=a\hat{\imath}+b\hat{\jmath}+c\hat{k}\Rightarrow a(x-x_0)+b(y-y_0)+c(z-z_0)=0$
with x_0,y_0,z_0 from P, Q, or R

2 Differential Equations

$\frac{dy}{dx}=x\to y=\frac{1}{2}x^2+C$
$\frac{dP}{dt}=kP\to P=P_0e^{kt} \quad k\in\mathbb{R}$
$\frac{dT}{dt}=k(T_0-T_E)\to T=(T_0-T_E)e^{kt}+T_E$
Product rule: $h(x)=f(x)g(x)\to h'(x)=f'(x)g(x)+f(x)g'(x)$
Quotient rule: $h(x)=\frac{f(x)}{g(x)}\to h'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$
Chain rule: $h(x)=f(g(x))\to h'(x)=f'(g(x))\cdot g'(x)$
Equivalently: $\frac{dz}{dx}=\frac{dz}{dy}\cdot\frac{dy}{dx}$
Separation of variables: $\frac{dy}{dx}=f(x)\cdot g(y)\to\frac{1}{g(y)}\,dy=f(x)dx$

2.1 Identities

$\frac{d}{dx}e^{f(x)}=f'(x)e^{f(x)} \qquad \frac{d}{dx}\ln f(x)=\frac{f'(x)}{f(x)}$
$\frac{d}{dx}a^x=a^x\ln a \qquad \frac{d}{dx}\log_a x=\frac{1}{x\ln a} \qquad x>0$

2.2 Partial Differential Equations: $f(x,y,z)$

Grad: $\nabla f=\frac{\partial f}{\partial x}\hat{\imath}+\frac{\partial f}{\partial y}\hat{\jmath}+\frac{\partial f}{\partial z}\hat{k} \qquad \nabla_{\vec{v}}f=\vec{v}\cdot\nabla f$
Div: $\nabla\cdot f=\frac{\partial f_x}{\partial x}+\frac{\partial f_y}{\partial y}+\frac{\partial f_z}{\partial z}$

$$\text{Curl: }\nabla\times f=\det\begin{bmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\ \frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\ f_x&f_y&f_z\end{bmatrix}$$

Chain rule: $z\equiv f(x(t),y(t)):\ \frac{dz}{dt}=\frac{\partial z}{\partial x}\frac{dx}{dt}+\frac{\partial z}{\partial y}\frac{dy}{dt}$
$z\equiv f(x_1(t_1,\ldots,t_m),\ldots,x_n(t_1,\ldots,t_m)):$
$\frac{\partial z}{\partial t_i}=\frac{\partial z}{\partial x_1}\frac{\partial x_i}{\partial t_i}+\cdots+\frac{\partial z}{\partial x_n}\frac{\partial x_n}{\partial t_i}$

3 Integration

By parts: $\int u\,dv=uv-\int v\,du$
or: $\int u\left(\frac{dv}{dx}\right)dx=uv-\int v\left(\frac{du}{dx}\right)dx$
Substitution rule: $\int f(u)u'\,dx=\int f(u)\,du$
Integrating factor given linear D. E., $\frac{dy}{dt}+P(t)y=Q(t)$:
$\mu(t)=e^{\int P(t)\,dt} \qquad \int\frac{d}{dt}(\mu(t)\cdot y)\,dt=\int\mu(t)Q(t)\,dt$

3.1 Identities

$\int e^x\,dx=e^x \qquad \int e^{-x}\,dx=-e^{-x} \qquad \int e^{ax}\,dx=\frac{1}{a}e^{ax} \qquad \int a^x\,dx=\frac{a^x}{\ln a}$

3.2 Applications

Disk, ↻x: $V=\pi\int_a^b[f(x)]^2dx$
$A_S=2\pi\int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$
Washer, ↻x: $V=\pi\int_a^b[f(x)]^2-[g(x)]^2\,dx$
Shell, ↻x: $V=2\pi\int_c^d yg(y)\,dy$
Shell, ↻y: $V=2\pi\int_a^b xf(x)\,dx$
$\ell_a=\int_{P_1}^{P_2}\sqrt{1+[f'(x)]^2}\,dx$
$x=f(t),\ y=g(t)\to\ \ell_a=\int_{P_1}^{P_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

$$A_S=\iint_S\sqrt{\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2+1}\,dxdy$$

3.3 Partial Fractions: $f(x)=P(x)/Q(x)$

$$Q(x):(ax+b)^k\to\sum_{i=1}^{k}\frac{A_i}{(ax+b)^i} \qquad (ax^2+bx+c)^k\to\sum_{i=1}^{k}\frac{A_ix+B_i}{(ax^2+bx+c)^i}$$

3.4 Line Integral

Line integral:
$\vec{F}=\langle F_1\hat{\imath},F_2\hat{\jmath},F_3\hat{k}\rangle \quad \vec{r}=\langle x\hat{\imath},y\hat{\jmath},z\hat{k}\rangle$

$$\int_C\vec{F}\cdot d\vec{r}=\int_{P_1}^{P_2}\vec{F}(\vec{r}(t))\cdot\vec{r}'\,dt=\int_C(F_1dx+F_2dy+F_3dz)$$

Conservative field, endpoints $P_1,\ P_2$:

$$\vec{F}=\nabla f \qquad \int_C\vec{F}\cdot d\vec{r}=\int_{P1}^{P2}\nabla f\cdot dr=f(P_2)-f(P_1)$$

$$\therefore\int_{C_n}\vec{F}\cdot d\vec{r}=\int_{C_{m\neq n}}\vec{F}\cdot d\vec{r}$$

Flux integral: $\Phi=\iint_S\vec{F}\cdot\hat{n}\,dS$

Divergence theorem: $\oiint_S\vec{F}\cdot\hat{n}\,dS=\iiint_V(\nabla\cdot\vec{F})\,dV$

Green's theorem:
$\vec{F}(x,y)=P(x,y)\hat{\imath}+Q(x,y)\hat{\jmath}$

$$\oint_C\vec{F}\cdot d\vec{r}=\iint_R\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dA$$

Stokes' theorem: $\oint_C\vec{F}\cdot d\vec{r}=\iint_S(\nabla\times\vec{F})\,dS$

4 Series

Arithmetic: $\sum_{k=0}^{n-1}a+kd=\frac{n}{2}(2a+(n-1)d)$

Geometric: $\sum_{k=0}^{n-1}ar^k=a\left(\frac{r^n-1}{r-1}\right)$

5 Statistics

$\bar{x}=\frac{\sum x_i}{n} \qquad \mu=\frac{\sum x_i}{N} \qquad s=\sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}} \qquad \sigma=\sqrt{\frac{\sum(x_i-\mu)^2}{N}}$
$r_{xy}=\frac{1}{n-1}\sum\frac{x_i-\bar{x}}{s_x}\frac{y_i-\bar{y}}{s_y}$
Sample geometric mean: $\sqrt[n]{\prod x_i}$ Sample RMS: $\sqrt{\frac{\sum x_i^2}{n}}$

5.1 Binomials

$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^kb^{n-k} \qquad \binom{n}{k}=\frac{n!}{k!(n-k)!}$
$n\in\mathbb{N} \quad k\in\mathbb{N}^* \quad k\leq n$

5.2 Probability and Combinatorics

$\mathbb{P}(A \vee B) = \mathbb{P}(A) + \mathbb{P}(B) \qquad \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

$\mathbb{P}(A \wedge B) = \mathbb{P}(A)\mathbb{P}(B) \quad \mathbb{P}(A \cap B) = \mathbb{P}(A|B)\mathbb{P}(B) = \mathbb{P}(B|A)\mathbb{P}(A)$

$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B|A)\mathbb{P}(A) + \mathbb{P}(B|\neg A)\mathbb{P}(\neg A)}$

Binomial distribution: k successes, n trials, success probability $\mathbb{P}$:

$B(k; n, \mathbb{P}) = \binom{n}{k}\mathbb{P}^k(1-\mathbb{P})^{n-k}$

${}^nP_k = n^k$ (repetition) ${}^nP_k = \frac{n!}{(n-k)!}$ (no repetition)

${}^nC_k = \frac{(n+k-1)!}{k!(n-1)!}$ (repetition) ${}^nC_k = \frac{n!}{k!(n-k)!}$ (no repetition)

Multisets: ${}^nP_r(n; n_1, n_2, \ldots, n_k) = \frac{n!}{n_1!n_2!\cdots n_k!}$

Derangement: $!n = n! \sum_{i=0}^{n} \frac{(-1)^i}{i!} \quad n \geq 0$

5.3 t-Statistic

One sample: $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$

Paired samples:

$$t = \frac{\sum(x_1 - x_1')}{n}\left[\frac{\sum(x_1-x_1')^2 - \frac{[\sum(x_1-x_1')]^2}{n}}{n(n-1)}\right]^{-1/2}$$

Independent samples:

$$t = (\mu_A - \mu_B)\left[\left(\frac{\sum(A^2) - \frac{(\sum A)^2}{n_A} + \sum(B^2) - \frac{(\sum B)^2}{n_B}}{n_A + n_B - 2}\right)\left(\frac{1}{n_A} + \frac{1}{n_B}\right)\right]^{-1/2}$$

5.4 Statistical Tests for Comparing Means

$\frac{s_1^2}{s_2^2} > \texttt{F.INV.RT}(\alpha, \nu_1, \nu_2) \Rightarrow$ significant

$s_m \sim s_n: \ t = \frac{|\bar{x}_1 - \bar{x}_2|}{s_{\text{pooled}}}\sqrt{\frac{n_1 n_2}{n_1+n_2}} \qquad s_{\text{pooled}} = \sqrt{\frac{s_1^2\nu_1 + s_2^2\nu_2}{\nu_1+\nu_2}}$

$s_m \nsim s_n: \ t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$

$\nu' = \frac{\left[s_1^2/n_1 + s_2^2/n_2\right]^2}{(s_1^2/n_1)^2/\nu_1 + (s_2^2/n_2)^2/\nu_2} \qquad t > \texttt{T.INV.2T}(\alpha, \nu') \Rightarrow$ significant

6 Geometry

$A_{\text{parallelogram}} = b\hbar \qquad A_{\text{triangle}} = 0.5b\hbar$

$A_{\text{trapezoid}} = 0.5(a+b)\hbar \qquad A_{\text{circle}} = \pi r^2$

$V_{\text{cube}} = s^3 \quad A_{s,\text{ cube}} = 6s^2$

$V_{\text{sphere}} = \frac{4}{3}\pi r^3 \quad A_{s,\text{ sphere}} = 4\pi r^2$

$V_{\text{rec. prism}} = \ell_1\ell_2\hbar \quad A_{s,\text{ rec. prism}} = 2(\ell_1\ell_2 + \ell_1\hbar + \ell_2\hbar)$

$V_{\text{cylinder}} = \pi r^2\hbar \quad A_{s,\text{ cylinder}} = 2\pi r\hbar + 2\pi r^2$

$V_{\text{rt. circ. cone}} = \frac{\pi r^2\hbar}{3} \quad A_{s,\text{ rt. circ. cone}} = \pi r\sqrt{r^2 + \hbar^2}$

$V_{\text{pyramid}} = \frac{\ell_1\ell_2\hbar}{3}$

$A_{s,\text{ pyramid}} = \ell_1\ell_2 + \ell_1\sqrt{\left(\frac{\ell_2}{2}\right)^2 + \hbar^2} + \ell_2\sqrt{\left(\frac{\ell_1}{2}\right)^2 + \hbar^2}$

7 Trigonometric Identities

$\sin\theta = (\csc\theta)^{-1} \qquad \cos\theta = (\sec\theta)^{-1} \qquad \tan\theta = (\cot\theta)^{-1}$

co-function$(\frac{\pi}{2} - \theta) =$ function(θ)

$\sin(u \pm v) = \sin u\cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

$\sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$

$\sin u + \sin v = 2\sin\frac{u+v}{2}\cos\frac{u-v}{2}$

$\sin u - \sin v = 2\cos\frac{u+v}{2}\sin\frac{u-v}{2}$

$\cos u + \cos v = 2\cos\frac{u+v}{2}\cos\frac{u-v}{2}$

$\cos u - \cos v = -2\sin\frac{u+v}{2}\sin\frac{u-v}{2}$

$\sin u \sin v = \frac{\cos(u-v) - \cos(u+v)}{2}$

$\cos u \cos v = \frac{\cos(u-v) + \cos(u+v)}{2}$

$\sin u \cos v = \frac{\sin(u+v) + \sin(u-v)}{2}$

$\cos u \sin v = \frac{\sin(u+v) - \sin(u-v)}{2}$

$\cos^2\theta + \sin^2\theta = 1 \qquad e^{\imath\pi} + 1 = 0 \qquad e^{\imath x} = \cos x + \imath\sin x$

$\cos x = \frac{e^{\imath x} + e^{-\imath x}}{2} \qquad \sin x = \frac{e^{\imath x} - e^{-\imath x}}{2}$

8 Fourier and Laplace Transforms

$$\mathscr{F}g(t) = \hat{g}(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi\imath ft}\,dt$$

$$\mathscr{F}^{-1}\hat{g}(f) = g(t) = \int_{-\infty}^{\infty} \hat{g}(f)e^{2\pi\imath ft}\,df$$

$$f(t) \sim A_0 + \sum_{n=1}^{\infty}\left[A_n\cos\left(n\frac{2\pi t}{T}\right) + B_n\sin\left(n\frac{2\pi t}{T}\right)\right]$$

$$A_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt$$

$$A_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\left(\frac{n2\pi t}{T}\right)\,dt \qquad B_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin\left(\frac{n2\pi t}{T}\right)\,dt$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{-n2\pi\imath t}{T}} \qquad c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{\frac{-n2\pi\imath t}{T}}dt \qquad n \in \mathbb{Z}$$

$\mathscr{L}g(t) = G(s) = \int_o^{\infty} g(t)e^{-st}\,dt \qquad s = \sigma + \imath\omega \qquad 0 < t < \infty$

$\mathscr{L}^{-1}G(s) = g(t) = \frac{1}{2\pi\imath}\int_{\sigma-\imath\infty}^{\sigma+\imath\infty} G(s)e^{st}\,ds$